Josephson-Junction Qubits

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- “Atom” based on nonlinear microwave resonator
- Scalable system using IC fabrication
Main Concepts

• Challenge: Coupling vs. Decoherence

• Qubits from E&M modes – non-linearity essential

• Josephson effect and non-linear inductance:
  Qubit formed by non-linear LC

• Noise model of decoherence (equivalent to Spin-Boson)

• Decoupling of qubits through impedance transformer circuits

• Promising experimental results (circuits work!)

• New decoherence mechanism:
  Need to improve on junction fabrication
Operations for Quantum Computation (DiVincenzo criteria)

Classical Computation:

- Initialize state

- Logic

  **not**

  \[ 0 \rightarrow 1 \]

  \[ 1 \rightarrow 0 \]

  **and**

  \[ 00 \rightarrow 0 \]

  \[ 01 \rightarrow 0 \]

  \[ 10 \rightarrow 0 \]

  \[ 11 \rightarrow 1 \]

- Output result

- Logic errors:
  Error correction possible

Quantum Computation:

- Initialize state \( \Psi_i = |000..0> \)

- Logic via series of operations:

  State

  \[ |0> \rightarrow |1> \]

  Manipulation

  \[ |1> \rightarrow |0> \]

  (1 qubit)

  \[ |0> \rightarrow (|0>+|1>)/2 \]

  Controlled not

  (2 qubit)

  \[ |00> \rightarrow |00> \]

  \[ |10> \rightarrow |10> \]

  \[ |01> \rightarrow |11> \]

  \[ |11> \rightarrow |01> \]

  \{ + linear superposition

  bit \[ \text{control} \]

- Final state measurement

  Measure qubits of state \( \Psi_f \)

- Coherence:

  \( \tau_{\text{coherence}} / \tau_{\text{logic}} \sim \text{number logic operations} \)
Challenge: **Coupling vs. Decoherence**

Experimental challenge:
Couple qubits to each other, control, & measure, not noise and dissipation
Experimental Systems

Atoms

- Ions
- Neutral Atoms
- NMR

Feynmann (1985): “it seems that the laws of physics present no barrier to reducing the size of computers until bits are the size of atoms, and quantum behavior holds sway.”

- Spin
- Semiconductor spin
- Quantum Dot
Feynmann (1985): “it seems that the laws of physics present no barrier to reducing the size of computers until bits are the size of atoms, and quantum behavior holds sway.”

Experimental Systems

**Atoms**
- Ions
- Neutral Atoms
- NMR
- Spin
- Semiconductor spin
- Quantum Dot

**EM modes**
- Photons
- Superconductor SET
- 2-Degenerate SSET
- 3 Junction SQUID
- RF SQUID
- Josephson Junctions (this work)

**Coherence easier**

**Coupling easier**

**Charge**
- (NEC, Chalmers)
- (Saclay)
- (Delft)
- (MIT)
- (NIST, Kansas)

**Flux**

**Phase**
Superconductivity

- **Phase degree of freedom**: $\phi$
  - Cooper pairs – states of zero momentum
  - Only remaining degree of freedom is $\phi$

- **Supercurrent**: $j_s \propto \nabla \phi$

- **Energy gap of excitations**: $\Delta \approx 1.76 kT_c$
  - No dissipation for $f < 720$ GHz (Nb)

- **Flux quantization**

- **Josephson effect**
  \[
  I \propto \left| \langle \Psi |Tc_{kL}c_{-kL}c_{kR}^{+}c_{-kR}^{+}\rangle \Psi \rangle \right|^2 - c.c.
  \approx e^{i\phi_L} e^{-i\phi_R} - c.c.
  = I_0 \sin(\phi_L - \phi_R)
  \]
  \[
  \phi_L \rightarrow \phi_L + \frac{2e}{\hbar} \int V dt
  \]
  - Large $\Delta$ gives little/no dissipation in junction from fabrication imperfections
Josephson-Junction Physics (classical)

\[ I_j = I_0 \sin \delta \]

\[ V = (\Phi_0 / 2\pi) \delta \]

\[ U = \int I_j V dt \]

\[ = \frac{I_0 \Phi_0}{2\pi} \int \sin \delta \frac{d\delta}{dt} dt \]

\[ = -\frac{I_0 \Phi_0}{2\pi} \cos \delta \]

\[ \left[ C \left( \frac{\Phi_0}{2\pi} \right)^2 \right] \ddot{\delta} + \left[ \frac{1}{R} \left( \frac{\Phi_0}{2\pi} \right)^2 \right] \dot{\delta} + \frac{\partial}{\partial \delta} \left[ -I_0 \frac{\Phi_0}{2\pi} \cos \delta - I \frac{\Phi_0}{2\pi} \delta \right] = 0 \]

mass damping potential \( U(\delta) \)

\[ 2\Delta_{sc} \sim 3 \text{mV} \]
Qubit: Nonlinear LC resonator

\[ I = I_0 \sin \delta \]
\[ V = \frac{\Phi_0}{2\pi} \dot{\delta} \]
\[ \dot{I}_j = I_0 \cos \delta \quad \delta \equiv \left(\frac{1}{L_j}\right)V \]
\[ L_j = \frac{\Phi_0}{2\pi I_0 \cos \delta} \]

**nonlinear inductor**

\[ \Delta U = \frac{4\sqrt{2} I_0 \Phi_0}{3 \cdot 2\pi} \left[ 1 - I/I_0 \right]^{3/2} \]
\[ \omega_p = \left( \frac{2\sqrt{2\pi}}{\Phi_0} \frac{I_0}{C} \right)^{1/2} \left[ 1 - I/I_0 \right]^{1/4} \]
\[ \gamma_{10} \approx \frac{1}{RC} \quad \text{Lifetime of state } |1> \]

\[ \langle V \rangle = 0 \]
\[ \langle V \rangle \approx 1 \text{mV} \]

\[ \frac{\Gamma_{n+1}}{\Gamma_n} \sim 1000 \]

\[ \omega_{10}, \omega_{21}, \omega_{32} \]
Qubit Taxonomy

Circuit elements:
- $I_0$
- $L$
- $C$

Hamiltonian:
- $-\frac{I_0 \Phi_0}{2\pi} \cos \delta$
- $-\frac{I \Phi_0}{2\pi} \delta$
- $(\Phi_0 \delta)^2 / 2L$
- $\frac{e^2}{2C} q^2$

Quantum mechanics:
- $[\hat{\Phi}, \hat{Q}] = i\hbar$
- $[\hat{\delta}, \hat{q}] = 2i$

<table>
<thead>
<tr>
<th>Phase</th>
<th>Flux</th>
<th>Charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-linearity</td>
<td>$I \rightarrow I_0$</td>
<td>$L \equiv L_{J0}$</td>
</tr>
<tr>
<td>$E_J / E_C = \frac{I_0 \Phi_0 / 2\pi}{e^2 / 2C}$</td>
<td>10³</td>
<td>1</td>
</tr>
<tr>
<td>Area ($\mu m^2$):</td>
<td>10-100</td>
<td>0.1-1</td>
</tr>
<tr>
<td>Potential &amp; wavefunction</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Engineering</td>
<td>$Z_J = 1/\omega_0 C$</td>
<td>10 $\Omega$</td>
</tr>
</tbody>
</table>
Josephson-Junction Qubit

- **State Preparation**
  Wait \( t > 1/\gamma_{10} \) for decay to \( |0> \)

- **Qubit logic with bias control**
  \[
  I = I_{dc} + \delta I_{dc}(t) + I_{\mu wc}(t) \cos \omega_{10}t + I_{\mu ws}(t) \sin \omega_{10}t
  \]

  \[
  H_{(2)} = \sigma_x \bullet I_{\mu wc} \bullet (\hbar / 2 \omega_{10} C)^{1/2} / 2
  \]

  \[
  + \sigma_y \bullet I_{\mu ws} \bullet (\hbar / 2 \omega_{10} C)^{1/2} / 2
  \]

  \[
  + \sigma_z \bullet \delta I_{dc}(t) \bullet (\partial E_{10} / \partial I_{dc}) / 2
  \]

- **State Measurement** (Junction acts as “photomultiplier”)
  \( |0> \): zero voltage
  \( |1> \): voltage

  \( \langle V \rangle = 0 \)

  \( \langle V \rangle = 1 \text{ mV} \)

  **I.** \( \omega_{21} \) microwave pulse

  **II.** I pulse (lower barrier)

  **Fidelity > 99%!!**

  With \( \Gamma_i / \Gamma_{i-1} \approx 1000 \)
Effective Hamiltonian \( \sigma \cdot B \)

\[
H = \frac{1}{2C} \hat{q}^2 + \frac{\Phi_0}{2\pi} \left[ -I_0 \cos \hat{\delta} - (I_{dc} + \Delta I) \hat{\delta} \right]
\]

\[
= \frac{1}{2C} \hat{q}^2 + H_{cubic}(I_{dc}, \hat{\delta}) - \frac{\Phi_0}{2\pi} \Delta I \hat{\delta}
\]

Solve numerically

\[\approx \begin{pmatrix} 0 & 0 \\ 0 & \hbar \omega_{10} \end{pmatrix} + \frac{\Phi_0}{2\pi} \Delta I \begin{pmatrix} \langle 0|\hat{\delta}|0 \rangle & \langle 0|\hat{\delta}|1 \rangle \\ \langle 1|\hat{\delta}|0 \rangle & \langle 1|\hat{\delta}|1 \rangle \end{pmatrix} \]

Perturbation

Basis transform (rotating frame):

\[V = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\omega_{10}t} \end{pmatrix}\]

\[\tilde{H} = V^+HV - i\hbar V^+ (\partial_t V)\]

\[
\tilde{H} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} + \frac{\Phi_0}{2\pi} \left( \Delta I_{dc} + I_{uwc} \cos \omega_{10} t + I_{uws} \sin \omega_{10} t \right) \begin{pmatrix} \langle 0|\hat{\delta}|0 \rangle & \langle 0|\hat{\delta}|1 \rangle e^{-i\omega_{10}t} \\ \langle 1|\hat{\delta}|0 \rangle e^{i\omega_{10}t} & \langle 1|\hat{\delta}|1 \rangle \end{pmatrix}
\]

\[\approx \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} + \frac{\Phi_0}{2\pi} \Delta I_{dc} \begin{pmatrix} \langle 0|\hat{\delta}|0 \rangle & 0 \\ 0 & \langle 1|\hat{\delta}|1 \rangle \end{pmatrix} + \frac{I_{uwc}}{2} \begin{pmatrix} 0 & \langle 0|\hat{\delta}|1 \rangle \\ \langle 1|\hat{\delta}|0 \rangle & 0 \end{pmatrix} + \frac{I_{uws}}{2} \begin{pmatrix} 0 & i\langle 0|\hat{\delta}|1 \rangle \\ -i\langle 1|\hat{\delta}|0 \rangle & 0 \end{pmatrix}
\]

Rotating wave approximation (neglect off resonant terms):

\[\sigma_z \quad \sigma_x \quad \sigma_y\]
Qubit Logic with Capacitive Coupling

\[ C_x \approx \frac{1}{300} C \]

\[ C_{\text{stray}} \sim 0.05-0.5 \text{ fF/µm} \]

\[ H_{\text{int}} \propto C_x q_1 q_2 \]
\[ \propto \sigma_{y_1} \sigma_{y_2} \]

Turn-off interaction with single qubit operations (e.g., NMR)

Modulate interaction by de-tuning resonance frequencies

Theory (Maryland): CNOT & Phase gates

Experiment (Maryland): Level splittings

Large junctions – \( C_{\text{stray}} \) unimportant
Coupling to more qubits, lower crosstalk
Josephson Junction Decoherence

\[ H_{(2)} = \sigma_x \cdot I_{\text{wc}}(t) \cdot (\hbar / 2\omega_{10}C)^{1/2} / 2 + \sigma_y \cdot I_{\text{ws}}(t) \cdot (\hbar / 2\omega_{10}C)^{1/2} / 2 + \sigma_z \cdot \delta I_{\text{dc}}(t) \cdot (\partial E_{10}/\partial I_{\text{dc}}) / 2 \]

\[ |0\rangle \rightarrow C_z \]

\[ |0\rangle + |1\rangle \rightarrow C_x \]

\[ \Psi = \cos \frac{\theta}{2} |0\rangle + \sin \frac{\theta}{2} e^{i\phi} |1\rangle \]
Josephson Junction Decoherence

Decoherence arises from noise and dissipation

\[ H_{(2)} = \sigma_x \cdot I_{\mu wc}(t) \cdot (\hbar / 2\omega_{10}C)^{1/2} / 2 \]
\[ + \sigma_y \cdot I_{\mu ws}(t) \cdot (\hbar / 2\omega_{10}C)^{1/2} / 2 \]
\[ + \sigma_z \cdot \delta I_{dc}(t) \cdot (\partial E_{10}/\partial I_{dc})/2 \]

Spontaneous Emission (decay 1 -> 0)

Noise at \( \omega_{10} \) (\( \sigma_x, \sigma_y \) op.’s)

Noise at LF (\( \sigma_z \) op.’s)
Decoherence from Dissipation (Spontaneous Emission)

\[ \gamma_{10} = \frac{\text{Re}\{1/Z(\omega_{10})\}}{C} \]

Dissipation at \( \omega_{10} \)
Noise and Spectral Density

\[ \langle I^2 \rangle = \int_0^\infty df S_I(f) \]

\[ \langle I(t)I(0) \rangle = \int_0^\infty df S_I(f) \cos(2\pi ft) \]

Resistor R
(low f) \[ S_I = 4kT/R \]
Decoherence from Noise at $\omega_{10}$ (Stimulated Emission)

\[ \gamma_s = \frac{1}{RC \exp(\hbar \omega_{10} / kT) - 1} \]

Exponentially small!
(Thermal eq. – non-trivial assumption)
Decoherence from LF Noise (Phase Noise)

\[
\phi_n(t) = \frac{\partial \omega_{10}}{\partial I} \int_0^t dt I_n(t)
\]

\[
\langle \phi_n^2(t) \rangle = \left( \frac{\partial \omega_{10}}{\partial I} \right)^2 \int_0^t dt \int_0^{t'} dt' \langle I_n(t) I_n(t') \rangle
\]

\[
= \left( \frac{\partial \omega_{10}}{\partial I} \right)^2 \int_0^\infty df S_I(f) \int_0^t dt \int_0^{t'} dt' \cos[2\pi f (t-t')]
\]

\[
= \left( \frac{\partial \omega_{10}}{\partial I} \right)^2 \int_0^\infty df S_I(f) \frac{\sin^2(\pi f t)}{(\pi f)^2}
\]

Equivalent to Spin-Boson calculations (PRB 67, 094510)

High frequency cutoff at \( f \sim 1/t \)
Josephson Junction Decoherence

\[ Z(\omega) = R \]

- Spontaneous Emission
  \[ \gamma_{10} = \frac{1}{RC} \]

- Phase Noise
  (No energy exchange!)

\[ \Psi(t) = |0\rangle + |1\rangle e^{i \int_0^t \omega_{10}[\Pi(t)] dt} \]

\[ = |0\rangle + |1\rangle e^{i \langle \omega_{10} \rangle t + i \phi_n(t)} \]

\[ \langle \phi_n^2(t) \rangle = \left( \frac{\partial \omega_{10}}{\partial I} \right)^2 \int_0^\infty \frac{4kT}{R} \frac{\sin^2(\pi ft)}{\pi f} df \]

\[ = \left( \frac{kT}{3\Delta U} \right) \frac{1}{RC} t \]

- 10 \, \mu s coherence times

- \( R \sim 10^6 \, \Omega \) at 5-10 GHz
  ~ \( 10^5 \, \Omega \) at 0-1 GHz

- Environment
Problem: Wire Impedances $\sim 100 \, \Omega$

$Z_L \sim Z_{\text{vac}} = 377 \, \Omega$

Embed `Josephson junction atom’ in an anti-cavity!
Broadband Impedance Transformers

Key concept: Power matching $I^2 R \sim \text{constant}$
Qubit operation $\rightarrow$ Measurement $\rightarrow$ Readout

$U(\delta)$

$\sim 5000$ states

"0"

"1"

$1 \Phi_0$

$\Phi_0$

I$_s$

I$_\phi$
Qubit operation → Measurement → Readout

Amplifier (and its dissipation!) turned on & off with $I_s$
- Adjustable $T_1$ -

$I_s \sim 0$
Balanced: no flux coupling
$R \sim 10^9 \ \Omega$

$I_s$ large
Imbalanced: flux measured
$R \sim 10^4 \ \Omega$

$U(\delta)$

SQUID flux

Switching current

$10 \ \mu A$

$1 \ \Phi_0$

Qubit Cycle

Qubit Op → Meas → Amp → Reset flux

Measure $p_1$
IC Fabrication

Al junction process & optical lithography

I_{\mu\text{wave}} \quad Qubit \quad I_s

100\mu m

I_\phi

qp trap for BE via junction qp trap for CE

AuCu Al SiO_2 Si/\text{SiO}_2 \text{ substrate}
Experimental Apparatus
Spectroscopy

\[ \omega \sim \omega_{10} \]

\[ I_\phi \]

\[ \omega/2\pi \text{ (GHz)} \]

\[ p_1 \]
Energy Levels vs. Bias Current

\[ \omega/2\pi \]

\( \omega_{10} \)

\( \omega_{21} \)

Increasing \( I \) (arb. Units)

\( p_1 = 0 \) : blue

\( p_1 = 1 \) : red
Rabi Oscillations

![Diagram of Rabi Oscillations]

(a) Occupation Probability of $|1\rangle$

(b) Occupation Probability of $|1\rangle$

(c) Occupation Probability of $|1\rangle$

(d) Rabi Frequency (MHz) vs. $\sqrt{\text{Power}}$ [a.u.]
Energy Relaxation

- Comparable with other experiments with Al junctions
- Energy relaxation time probably limited by flux bias $R$
Flux Qubit (Delft)
Large 1/f charge noise requires operation at degeneracy point \((d\omega/dq = 0)\).
Resonances & Rabi Oscillations

Rabi oscillations disappear at spurious resonances

$p_1=0 : \text{blue}$

$p_1=1 : \text{red}$

Decrease in coherence amplitude, not coherence time
Amplitude Decoherence

• Need to report amplitude of oscillations (visibility)

<table>
<thead>
<tr>
<th></th>
<th>barrier</th>
<th>coh. time</th>
<th>coh. ampl.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saclay</td>
<td>AlOx</td>
<td>1 µs</td>
<td>(30%)</td>
</tr>
<tr>
<td>Delft</td>
<td>AlOx</td>
<td>150 ns</td>
<td>50%</td>
</tr>
<tr>
<td>NIST</td>
<td>AlOx</td>
<td>41 ns</td>
<td>30%</td>
</tr>
<tr>
<td>NIST</td>
<td>NbAlOx</td>
<td>20 ns</td>
<td>15%</td>
</tr>
<tr>
<td>Kansas</td>
<td>AlN</td>
<td>5 µs</td>
<td>1%</td>
</tr>
</tbody>
</table>

• All experiments have reduced amplitude

• Spurious resonances observed in Saclay, Delft experiments
  (Small junctions: fewer resonances, but greater effect)

• Decoherence likely from small (unobserved) resonances

• Resonances are major source of decoherence?!
What causes extra resonances?

• Level-repulsion: Uncontrolled “Coupled qubit”

• Frequency shifts rule-out macroscopic EM modes

• Model as modulation of $I_0$ from resonant defect motion

\[ H_{\text{int}} = \left( -\frac{\Phi_0 I_{0A}}{2\pi} \cos \delta \right) \otimes |\Psi_A \rangle \langle \Psi_A | + \left( -\frac{\Phi_0 I_{0B}}{2\pi} \cos \delta \right) \otimes |\Psi_B \rangle \langle \Psi_B | \]

\[ = \frac{\Delta I_0}{2} \sqrt{\frac{\hbar}{2\omega_{10} C}} \left( |0 \rangle \langle 1| \otimes \langle e | g \rangle + |1 \rangle \langle 0| \otimes \langle g | e \rangle \right) \]

\[ |e \rangle = 2^{-1/2} \left( |\Psi_A \rangle - |\Psi_B \rangle \right) \]

\[ |g \rangle = 2^{-1/2} \left( |\Psi_A \rangle + |\Psi_B \rangle \right) \]

Explains energy-level repulsion
**IV’s:**

- 32 \( \mu \text{m}^2 \) junction
  - 25 MHz splittings \( (\Delta I_0 \sim 6 \times 10^{-5} I_0) \)
  - 1 resonance / 60MHz

- 0.1 \( \mu \text{m}^2 \) junction
  - \( \Delta I_0 \sim 10^{-4} I_0 \)
  - 1 trap / decade freq.

**Decoherence:**

- \( \tau \sim 4 \times 10^{-3} \)
- \( N_{\text{ch}} \sim 130000 \)
- 1 channel / (16nm)²
- \( \Delta I_0/I_0 \sim 1/N_{\text{ch}} \sim 7 \times 10^{-6} \)

\[ G_N = \frac{2e^2}{h} \sum \tau_i = \frac{2e^2}{h} N_{\text{ch}} \tau \]
\[ G_{S2} \approx \frac{2e^2}{h} \sum \tau_i^2 \quad (\Delta < eV < 2\Delta) \]

Assuming resonances & traps turn on/off channels:

- 8 channels / res.
- 5 res. / dec.-fr. - \( \mu \text{m}^2 \)
- 0.04 channels / trap
- 10 traps / dec.-fr. - \( \mu \text{m}^2 \)

**1/f Noise**

- See individual traps in sub-micron junctions
  (Savo, Wellstood, Clarke):

Resonances and 1/f noise are same phenomenon
Decoherence & Materials

All oxide tunnel barriers give similar $1/f$ noise

<table>
<thead>
<tr>
<th>Barrier</th>
<th>$S_{I0}^{1/2}(1\text{Hz})A^{1/2}/I_0$ (µm pA/Hz$^{1/2}$/µA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al-AlOx-Al</td>
<td>(~5)</td>
</tr>
<tr>
<td>Nb-AlOx-Nb</td>
<td>7-20</td>
</tr>
<tr>
<td>Nb-Ox-PbIn</td>
<td>7-20</td>
</tr>
<tr>
<td>Nb-NbOx-PbInAu</td>
<td>8</td>
</tr>
<tr>
<td>PbIn-Ox-Pb</td>
<td>15</td>
</tr>
<tr>
<td>NbN-AlN-NbN (epi)</td>
<td>1000</td>
</tr>
</tbody>
</table>

Need Materials Research – Qubits have vastly different requirements

Past Research: High $T_c$, $Q < 1$, low leakage junctions  
Qubits: $T_c > 1\text{K}$, leakage tolerated 
low fluctuations, low dielectric loss

Our research directions:
Barrier uniformity, barrier materials (AlN), epitaxial growth 
Large-area qubits are ideal test circuits
MBE Growth of Al on Si(111)

- Si(111)-(7×7)
  Flash anneal 1250°C
  Ordered

- Al seed layer 50 Å
  Evap. 100 K
  Anneal 450 K
  Ordered

- Al homo-epitaxy
  Evap. 300K
  Anneal 450 K
  Ordered

- Oxidation
  10T, 10 min, 300K
  Amorphous

- Al counter-electrode
  Amorphous
Main Concepts

- Challenge: Coupling vs. Decoherence
- Qubits from E&M modes – non-linearity essential
  - Josephson effect and non-linear inductance: Qubit formed by non-linear LC
- Noise model of decoherence (equivalent to Spin-Boson)
- Decoupling of qubits through impedance transformer circuits
- Promising experimental results (circuits work!)
- New decoherence mechanism:
  Need to improve on junction fabrication
Measurement X-coupling

\[ U(\delta) \sim 5000 \text{ states} \]

1 \Phi_0

What is quantum \( S_{ix}(\omega_{10}) \) ?

Drives 0 \rightarrow 1 \text{ transitions}

\[ P_{\text{error}} \sim 1000 \left( \frac{C_x}{C} \right)^2 \]

Rounding from tunneling, decay
Why is Quantum Computing Useful?

- Parallel computation of exponentially-large states
- Factorization of large numbers (Shor) Exponential speedup of algorithm
- Fast search algorithms (Grover) \( n^{1/2} \) vs. \( n \)
- Adiabatic algorithms for minimization (Farhi)
- Simulation of quantum systems (Feynman)
- Other? (Quantum Information Theory)